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260. Proposed by O. E. GLENN, Ph. D., Springfield, Mo.

The necessary and sufficient condition that a binary form be a perfect n th power is that its Hessian vanish.

I. Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Denoting $\frac{\partial u}{\partial x}$ by p , $\frac{\partial u}{\partial y}$ by q , the vanishing of the Hessian shows that $p=f(q)$, i. e., $q=mp$, since both p and q are homogeneous and of the same degree. By Lagrange's method of solving partial differential equations, we have

$$\frac{dx}{m} = \frac{dy}{-1} = \frac{du}{0}.$$

Hence, $u=\text{constant}$, $x+my=\text{constant}$, and a general solution is given by

$$u=f(x+my)=(x+my)^n,$$

since u is homogeneous in x, y . It is easily verified that when $u=(x+my)^n$ the Hessian vanishes. Hence this condition is both necessary and sufficient.

II. Solution by the PROPOSER.

A slightly different point of view from the above is afforded by the following method:

The Hessian is the Jacobian of the first derivatives p and q . Hence $p-mq=0$. Also $xp+yq=nu$, n being the order of u . Solving for p and q ,

$$p=\frac{nm u}{y+mx}, \quad q=\frac{nu}{y+mx}.$$

Also, $du=pdx+qdy=nu\frac{dy+mdx}{y+mx}$, or $\frac{du}{u}=n\frac{d(y+mx)}{y+mx}$.

Hence, $\log u=n \log k(y+mx)$, $u=(a_1x+a_2y)^n$.

261. Proposed by REV. R. D. CARMICHAEL, Hartselle, Ala.

Sum to infinity the series, $\frac{1}{n^p} + \frac{3}{n^{2p}} + \frac{5}{n^{3p}} + \frac{7}{n^{4p}} + \frac{9}{n^{5p}} + \dots$

Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Denoting n^{-p} by x , we have

$$\begin{aligned} \sum_{i=1}^{\infty} \frac{(2i-1)}{n^{ip}} &= x[1+3x+5x^2+7x^3+\dots] \\ &= x \sum (2r+1)x^r = 2x \sum rx^r + x \sum x^r \end{aligned}$$